University of Messina - Ph.D. Program in Mathematics

Variational and Non-Variational Methods

(Prof. Filippo D. Cammaroto)

(15 hours)

1. Gâteaux Differentiable Functionals on Banach Spaces

Gâteaux differentiable functionals defined on Banach spaces. Gâteaux derivative and its fundamental properties. Necessary conditions for minima and existence results. Functionals associated with differential problems. Preliminary tools from nonlinear analysis.

2. A General Variational Principle and Applications

Parameter-dependent minimization problems. Functionals of the form $\Phi+\lambda\Psi$. A general variational principle. Conditions ensuring the existence of critical points and multiplicity results. Some applications to nonlinear elliptic problems. Existence of infinitely many solutions for a two-point boundary value problem with homogeneous Dirichlet boundary conditions.

3. Variational Inequalities

Origins of the theory of variational inequalities. The formulations introduced by Stampacchia. Variational inequalities in Hilbert spaces. Fundamental existence and uniqueness theorems. Boundary value problems formulated as variational inequalities. Some applications to nonlinear analysis.

References

- **I. Ekeland, R. Témam**, *Convex Analysis and Variational Problems*, Classics in Applied Mathematics, SIAM, 1987
- **D. Kinderlehrer, G. Stampacchia**, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, New York, 1980
- **B. Ricceri**, *A General Variational Principle and Some of Its Applications*, *Journal of Computational and Applied Mathematics* **113** (2000), 401–410