University of Messina - Bachelor's Degree in Physics

Mathematics II

(Prof. Filippo D. Cammaroto)

Academic Year 2025/2026

1. Functions of Several Variables

Elements of topology in $\mathbb{R}^n$: open sets, closed sets, accumulation points. Continuity of functions of several real variables. Limits of functions of several variables. Domain of a function of several variables. Graph of a function of several variables: level curves and level surfaces. Partial derivatives. Schwarz’s Theorem. Derivative along a vector and directional derivative. Differentiable functions. Necessary conditions for differentiability. Differentiability of C^1 functions. Tangent plane and normal line to the graph of a function. Derivative of composite functions. Functions with zero gradient. Implicit functions. Dini’s Theorem. Derivative of the implicit function. Dini’s Theorem for systems of implicit functions. Local maxima and minima of functions of several variables. Quadratic forms. Fermat’s Theorem. Search for relative extrema by studying the Hessian matrix. Absolute maxima and minima of a function of several variables. Weierstrass Theorem. Constrained maxima and minima. Method of Lagrange multipliers.

2. Curves and Line Integrals

Vector-valued functions. Curves in $\mathbb{R}^3$. Parametric and vector equation of a curve. Regular and piecewise regular curves. Tangent vector to a curve at a point. Diffeomorphisms and reparametrizations. Rectifiable curves and length of a curve. Arc length. Curvature and torsion of a curve. Osculating plane. Frenet frame. Frenet equations. Fundamental theorem of space curves. Line integrals of functions with respect to arc length. Geometric meaning and properties of line integrals of functions. Applications of line integrals of functions to wire-like bodies: computation of mass, coordinates of the center of mass, and moment of inertia with respect to an oriented axis for a wire-like body with given density.

3. Differential Forms and Their Line Integrals

Linear differential forms and the associated vector fields. Line integrals of differential forms. Properties of line integrals of differential forms. Exact differential forms: conservative vector fields. Potential of a conservative field. Line integral of an exact differential form. Closed differential forms: irrotational vector fields. Differential forms on simply connected sets. Methods for computing the potential of a conservative vector field.

4. Multiple Integrals

Double integrals: Normal domains in $\mathbb{R}^2$. Measure of normal domains in $\mathbb{R}^2$. Decompositions of normal domains in $\mathbb{R}^2$. Riemann sums of a bounded function with respect to a decomposition of the domain. Riemann double integral of a bounded function defined on a normal domain in $\mathbb{R}^2$. Regular domains in $\mathbb{R}^2$. Double integrals on regular domains in $\mathbb{R}^2$. Geometric meaning of the double integral: volume of the cylindroid determined by a function of two variables. Reduction formulas for double integrals on normal domains. Change of variables in double integrals. Polar coordinates. Applications of double integrals: computation of areas, volumes, mass, center of mass, and moment of inertia with respect to an axis of a lamina. Gauss–Green Theorem in the plane. Applications of the Gauss–Green Theorem: area of a plane figure, divergence theorem in the plane, Stokes’ theorem in the plane.

Triple integrals: Normal domains in $\mathbb{R}^3$. Measure of normal domains in $\mathbb{R}^3$. Decompositions of normal domains in $\mathbb{R}^3$. Riemann sums of a bounded function with respect to a decomposition of the domain. Riemann triple integral of a bounded function defined on a normal domain in $\mathbb{R}^3$. Regular domains in $\mathbb{R}^3$. Triple integrals on regular domains in $\mathbb{R}^3$. Integration by projections and sections. Reduction formulas for triple integrals on normal domains. Triple integrals by layers and by fibers. Change of variables in triple integrals. Spherical coordinates. Cylindrical coordinates. Applications of triple integrals: computation of volumes, mass, center of mass, and moment of inertia with respect to an axis of a solid body.

Theoretical aspects: Necessary and sufficient conditions for integrability. Mean Value Theorem. Properties of the Riemann integral.

5. Surfaces and Surface Integrals

Surfaces in $\mathbb{R}^3$. Parametric equations of a surface. Regular surfaces. Tangent plane and normal line to a regular surface. Area of a regular surface. Surface integrals. Solids of revolution. Area of a surface of revolution. Volume of a solid of revolution. Pappus–Guldinus Theorems. Divergence Theorem. Stokes' Theorem. Solenoidal vector fields. Vector potential of a vector field. Computation of a solenoidal vector field from its curl. Flux of a solenoidal vector field through regular surfaces.

Recommended Textbooks

- R. A. Adams, C. Essex, *Calculus: A Complete Course*, Vols. 1–2, Pearson
- N. Fusco, P. Marcellini, C. Sbordone, *Lezioni di Analisi Matematica due*, Zanichelli.
- C. D. Pagani, S. Salsa, *Analisi Matematica II*, Masson.
- S. Salsa, A. Squellati, *Esercizi di Analisi Matematica II*, Masson.