University of Messina - Pd.D. Program in Mathematics and Computational Sciences

Operator Inclusions and Applications

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(30 ore)

1. Theory of Set-Valued Mappings

Basic definitions. Graph of a set-valued mapping. Inverse set-valued mapping. Selections of a set-valued mapping. Lower and upper inverse images. Distance from a set and set dilations. Fundamental properties of convex-valued set-valued mappings.

2. Semicontinuous and Measurable Set-Valued Mappings

Lower and upper semicontinuous set-valued mappings. Characterizations of semicontinuity. Properties of the graph of a set-valued mapping. Measurable and weakly measurable set-valued mappings. Measurable graph theorem. Kuratowski–Ryll-Nardzewski measurable selection theorem. Castaing representation theorem.

3. Fixed Point Theorems for Set-Valued Mappings

Fixed points of set-valued mappings. Kakutani's fixed point theorem. Ky Fan's fixed point theorem. Applications of fixed point theorems to the theory of operator inclusions.

4. Ricceri's Operator Inclusion Theorem

Superposition operators associated with set-valued mappings. Operator inclusions of the form

$$\Phi(u)(\omega)\in F(\omega,\Psi(u)(\omega)).$$

Statement and proof of Ricceri's Existence Theorem for Operator Inclusions. Role of measurability, lower semicontinuity and convexity assumptions. Main corollaries and applications.

5. Applications to Second-Order Boundary Value Problems

Boundary value problems for second-order ordinary differential inclusions. Operator formulation of the problem. Green's operator. Application of Ricceri's theorem to the existence of solutions. Differential inclusions with homogeneous Dirichlet boundary conditions. Examples and existence results.

6. Applications to Implicit Integral Equations

Integral operators and their fundamental properties. Implicit integral equations with discontinuous right-hand side. Operator formulation of the problem by means of operator inclusions. Application of Ricceri's theorem to the existence of essentially bounded solutions of implicit integral equations.

References

- **J.P. Aubin, H. Frankowska,** *Set-Valued Analysis*, Birkhäuser, Boston, 1990.
- **J.P. Aubin, A. Cellina,** *Differential inclusions*, Springer-Velag, Berlin, 1984.
- **C. Himmelberg,** *Measurable relations,* Fundamenta Mathematicae, 87(1):53–72, 1975.
- **K. Ryll-Nardzewski, C. Kuratowski,** *A general theorem on selectors,* Sci. Ser. Sci. Math. Astronom Phys, 13:397–403, 1965.

- **C. Castaing, M. Valadier,** *Measurable multifunctions,* Springer, 1977.
- **K. Fan,** *Fixed-point and minimax theorems in locally convex topological linear spaces,* Proceedings of the National Academy of Sciences of the United States of America, 38:121–126, 1952.
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- **O. Naselli Ricceri, B. Ricceri,** *An existence theorem for inclusions of the type $\psi(u)(t) \in F(t,\phi(u)(t))$ and application to a multivalued boundary value problem*. Applicable Analysis, 38(4):259–270, 1990.
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