University of Messina - Ph.D. Program in Mathematics

Elements of Functional Analysis and Applications

(Prof. Filippo D. Cammaroto)

(10 hours)

1. Ekeland's Variational Principle

Review of the theory of complete metric spaces. Lower semicontinuous functions and their characterizations. Ekeland's Variational Principle: statement and proof. Geometric interpretation of Ekeland's Variational Principle. First applications to nonlinear analysis. Caristi's Fixed Point Theorem.

2. Gâteaux Differentiable Functionals on Banach Spaces

Gâteaux differentiable functionals defined on Banach spaces. Gâteaux derivative and its fundamental properties. Necessary conditions for minima and existence results. Applications of Ekeland's Variational Principle to Gâteaux differentiable functionals.

3. A Complement to Ekeland's Variational Principle

A complement to Ekeland's Variational Principle. Applications to Caristi's Fixed Point Theorem and to Gâteaux differentiable functionals. Extensions and possible developments.

References

- I. Ekeland, R. Témam, *Convex Analysis and Variational Problems*, Classics in Applied Mathematics, SIAM, 1987.
- D. G. de Figueiredo, *Lectures on the Ekeland Variational Principle with Applications and Detours*, Tata Institute of Fundamental Research, Bombay, 1989.
- F. Cammaroto, A. Chinnì, *A Complement to Ekeland's Variational Principle in Banach Spaces*, *Bulletin of the Polish Academy of Sciences. Mathematics* **44** (1), 29–33, 1996.