



University of Messina - Ph.D. Program in Mathematics

Nonlinear Analysis

(Prof. Filippo D. Cammaroto)

(30 hours)

1. Ekeland's Variational Principle

Review of the theory of complete metric spaces. Lower semicontinuous functions and their characterizations. Ekeland's Variational Principle: statement, proof and geometric interpretation. First applications to nonlinear analysis. Caristi's Fixed Point Theorem.

2. Gâteaux Differentiable Functionals on Banach Spaces

Gâteaux differentiable functionals defined on Banach spaces. Gâteaux derivative and its fundamental properties. Necessary conditions for minima and existence results. Functionals associated with differential problems. Applications of Ekeland's Variational Principle to Gâteaux differentiable functionals.

3. A Complement to Ekeland's Variational Principle

A complement to Ekeland's Variational Principle. Applications to Caristi's Fixed Point Theorem and to Gâteaux differentiable functionals. A Liouville-type theorem for harmonic functions on exterior domains. A Liouville-type theorem for the homogeneous wave equation. Extensions and possible developments.

4. Variational Inequalities

Origins of the theory of variational inequalities. The formulations introduced by Stampacchia. Variational inequalities in Hilbert spaces. Fundamental existence and uniqueness theorems. Boundary value problems formulated as variational inequalities. Some applications to nonlinear analysis.

5. A General Variational Principle and Applications

Parameter-dependent minimization problems. Functionals of the form $\Phi + \lambda\Psi$. A general variational principle. Conditions ensuring the existence of critical points and multiplicity results. Some applications to nonlinear elliptic problems. Existence of infinitely many solutions for a two-point boundary value problem with homogeneous Dirichlet boundary conditions.

References

- **I. Ekeland, R. Témam**, *Convex Analysis and Variational Problems*, Classics in Applied Mathematics, SIAM, 1987
- **D.G. De Figueiredo**, *Lectures on The Ekeland Variational Principle with Applications and Detours*, Tata Institute of Fundamental Research, Bombay, 1989
- **D. Kinderlehrer, G. Stampacchia**, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, New York, 1980

- **B. Ricceri**, *A general variational principle and some of its applications*, Journal of Computational and Applied Mathematics 113, (2000), 401–410
- **F. Cammaroto, A. Chinnì**, *A Complement to Ekeland's Variational Principle in Banach Spaces*, Bulletin of the Polish Academy of Sciences. Mathematics, 44 (1), 29-33, 1996.
- **F. Cammaroto, A. Chinnì**, *A Liouville-type Theorem for Harmonic Functions on Exterior Domains*, Journal of Mathematical Analysis and Applications, 244 , 1-9, 2000.
- **F. Cammaroto, A. Chinnì**, *A Liouville-type Theorem for the Homogeneous Wave Equation*, Le Matematiche, LVII (I), 167-170, 2002.
- **F. Cammaroto, A. Chinnì**, *Infinitely many solutions for a two points boundary value problem*, Far East Journal of Mathematical Sciences, 11 (1), 41-51, 2003.